Solutions to JEE MAIN – 6 |JEE - 2024

PHYSICS

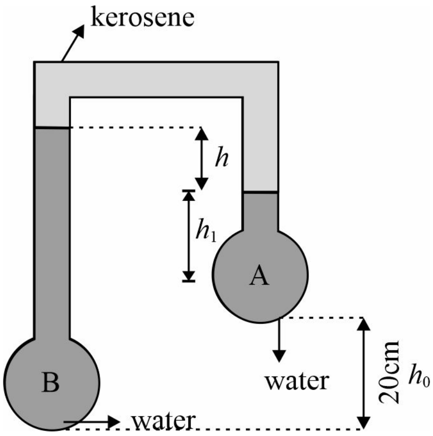
SECTION-1

1.(A) Using continuity equation, $A_1 v_1 = A_2 v_2$

$$\frac{v_1}{v_2} = \frac{A_2}{A_1} = \frac{\pi r_2^2}{\pi r_1^2} = \left(\frac{r_2}{r_1}\right)^2 = \frac{4}{1}$$

2.(A) $P_A - \rho g h_1 - \rho_k g h + \rho g h + \rho g h_1 + \rho g h_0 = P_B$

$\Rightarrow \quad (\rho - \rho_k) g h = P_B - P_A - \rho g 0.2$



$\Rightarrow \quad 2000h = 3000 - 2000$

$\Rightarrow \quad h = 0.5m \qquad \therefore \qquad h = 50cm$

3.(C) $M_p = \dfrac{M_e}{1000}, \ R_p = \dfrac{R_e}{8};\qquad W_e = \dfrac{GmM_e}{R_e^2}, W_p = \dfrac{GmM_p}{R_p^2} = \dfrac{\frac{GmM_e}{1000}}{\left(\frac{R_e}{8}\right)^2} = \dfrac{64}{1000} \times \dfrac{GmM_e}{R_e^2}$

$\Rightarrow \quad W_p = \dfrac{64}{1000} \times W_e = \dfrac{64}{1000} \times 500\ N = 32\ N$

4.(B) A to B : $\dfrac{1}{2}mV^2 + \dfrac{1}{2}.\dfrac{mR^2}{2}.\dfrac{V^2}{R^2} = \dfrac{mgh}{2} \Rightarrow \dfrac{3}{4}mV^2 = \dfrac{mgh}{2}$

$\Rightarrow \quad KE_{\text{Rotational}} = \dfrac{1}{2}.\dfrac{mR^2}{2}\dfrac{V^2}{R^2} = \dfrac{mV^2}{4}$

B to C : $\dfrac{1}{2}mV_c^2 = \dfrac{1}{2}mV^2 + \dfrac{mgh}{2} = \dfrac{1}{2}mV^2 + \dfrac{3}{4}mV^2 = \dfrac{5}{4}mV^2$

Ratio $= \dfrac{KE_{\text{trans}}}{KE_{\text{Rotational}}} = \dfrac{\frac{5}{4}mV^2}{\frac{1}{4}mV^2} = 5:1$

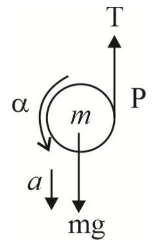
5.(B) $I = \dfrac{mL^2}{12} + \dfrac{mL^2}{3}\left(\sin^2 30°\right) \times 2 \ = \dfrac{mL^2}{12} + \dfrac{mL^2}{6} = \dfrac{mL^2}{4}$

6.(B) Instantaneous axis of rotation passes through point P

$\therefore \quad \tau_p = I_p \alpha$

$\Rightarrow \quad mg.R = 2mR^2\alpha$

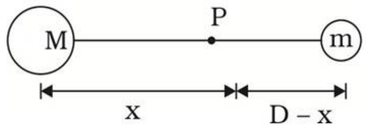
$\therefore \quad a = \alpha R = \frac{g}{2}$



7.(C) $a = \frac{g\sin\theta}{1+\frac{k^2}{R^2}} = \frac{g\sin\theta}{1+\frac{2}{5}} = \frac{5}{7}g\sin\theta;$ $\quad a_{\text{smooth}} = g\sin\theta = \frac{7a}{5}$

8.(B) $\frac{100GM}{x^2} = \frac{GM}{(D-x)^2}$

$\frac{10}{x} = \frac{1}{(D-x)};\ D = \frac{11}{10}x;\quad x = \frac{10}{11}D$

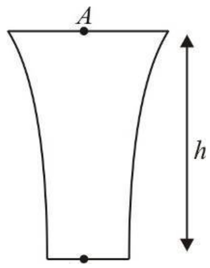


9.(B) $AV_0 = A/4 \times V_1 \quad \Rightarrow \quad V_1 = 4V_0$

$P + \frac{1}{2}\rho v_1^2 = P + \rho gh + \frac{1}{2}\rho v_0^2$

$\Rightarrow \quad v_1^2 = v_0^2 + 2gh;\ 16v_0^2 = v_0^2 + 2gh$

$h = \frac{15v_0^2}{2g}$



10.(B) $\frac{h_2}{h_1} = \frac{\sigma_2\cos\theta_2}{\rho_2} \times \frac{\rho_1}{\sigma_1\cos\theta_1} \quad \frac{h_2}{h_1} = \frac{210\times\frac{\sqrt{3}}{2}}{2} \times \frac{1}{70\times 1} \Rightarrow h_2 = \frac{h_1\times 3\sqrt{3}}{4} = \frac{3\times 3\sqrt{3}}{2} = 9\sqrt{3}/2$

11.(D) $\frac{r_{\min}}{r_{\max}} = \frac{\sqrt{3}}{2}$

Angular momentum is conserved:

$mv_0r_{\min} = mv'r_{\max};\quad v' = v_0 \times \frac{r_{\min}}{r_{\max}} = \frac{v_0\sqrt{3}}{2}$

12.(B) For a satellite of mass *m* rotating in a circular orbit of radius *R* around a planet of mass *M*, kinetic energy is given by $T = \frac{GMm}{2R}$

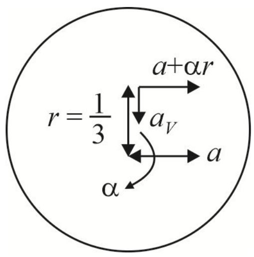
Here, $T_A = \frac{GMm}{2R}$ and $T_B = \frac{GM(2m)}{2(2R)} = \frac{GMm}{2R};\quad \frac{T_A}{T_B} = 1$

13.(D) $\alpha = \frac{a}{r} = 3rad/s^2$

$a_H = a + \alpha r = 3 + 3.\frac{1}{3} = 4m/s^2$

$a_V = \omega^2 r = \frac{V^2}{R^2}.r = 3m/s^2$

$\therefore \quad$ Resultant acc. $= \sqrt{4^2+3^2} = 5\text{m/s}^2$



14.(B) Here, $u = V_0$, $\omega_0 = -\dfrac{V_0}{2R}$

Let F_t be the friction force acting in backward direction.

$$a = -\frac{F_f}{m}, \ \alpha = \frac{\tau}{I} = \frac{F_f R}{mR^2}$$

At pure rolling : $V = V_0 - \left(\dfrac{F_f}{m}\right)t$ and $\dfrac{V}{R} = -\dfrac{V_0}{2R} + \left(\dfrac{F_f}{mR}\right)t$ (in pure rolling, $V = R\omega$)

$$\Rightarrow \quad V_0 - V = V + \frac{V_0}{2}; \quad 2V = \frac{V_0}{2} \Rightarrow V = \frac{V_0}{4}$$

15.(C) $6\pi\eta r V_t = (d-\rho)Vg$; $V_t = \dfrac{(d-\rho)Vg}{6\pi\eta r}$

$$\Rightarrow \quad V_t \alpha r^2; \qquad \frac{V_{t_1}}{V_{t_2}} = \left(\frac{r_1}{r_2}\right)^2 = \frac{1}{16} \quad \Rightarrow \quad V_{t_2} = 16V_{t_1}$$

16.(B) $v = \sqrt{\dfrac{2gh}{1+\dfrac{k^2}{R^2}}} = \sqrt{\dfrac{(2)(10)(7)}{1+\dfrac{2}{5}}}\ m/s = 10m/s$

17.(C) From conservation of angular momentum about hinged point

$$mvl = \left(\frac{ml^2}{3} + 2ml^2\right)\omega; \qquad \omega = \frac{3}{7}.\frac{v}{l}$$

Now from conservation of energy

$$\frac{1}{2} \times \frac{7}{3} ml^2 \times \omega^2 = mgl + 2mg.2l; \qquad v = \sqrt{\frac{70}{3} gl}$$

18.(A) Let *v* be the velocity of the body when it escapes the gravitational pull of the earth and *u* be the velocity of projection of the body from the earth's surface. Then by the law of the conservation of energy

$$\frac{1}{2}mu^2 - \frac{GMm}{R} = \frac{1}{2}mv^2 + 0$$

Where m and M are the masses of the body and earth respectively and R is the radius of the earth.

$$v^2 = u^2 - \frac{2GM}{R} = u^2 - v_e^2 \qquad \left(\because \ v_e = \sqrt{\frac{2GM}{R}}\right)$$

$$\Rightarrow \quad v^2 = \sqrt{(2\times 11.2)^2 - (11.2)^2} \qquad (\because \ v_e = 11.2 \text{ km s}^{-1})$$

$$\Rightarrow \quad v = \sqrt{3} \times 11.2 \text{ km s}^{-1}$$

19.(C) $K = \dfrac{1}{2} I_1 \omega_1^2$; By conservation of angular momentum

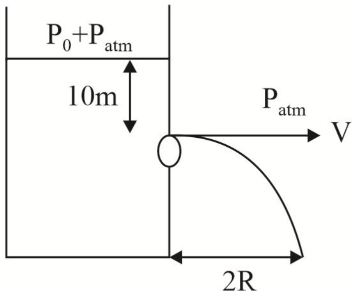
$I_1\omega_1 = I_2\,\omega_2$ and $I_2 = 3I_1$ (given) $\Rightarrow$ $\omega_2 = \dfrac{\omega_1}{3}$

$$K_2 = \frac{1}{2} I_2 \omega_2^2 = \frac{1}{2}(3I_1)\left(\frac{\omega_1}{3}\right)^2 = \frac{1}{6} I_1 \omega_1^2 = K/3$$

20.(D) $\rho = 10^3 kg / m^3$

$h = 10m$

$R = \sqrt{2gh}.t$ ...(1)



$P_0 + P_{atm} + \rho gh = P_{atm} + \frac{1}{2}\rho V^2$

$\therefore \quad V = \sqrt{\frac{2P_0}{\rho} + 2gh}$

$2R = Vt$ ...(2)

$\Rightarrow \quad 2\sqrt{2gh} = \sqrt{\frac{2P_0}{\rho} + 2gh}$

$\therefore \quad P_0 = 3\rho gh = 3atm$

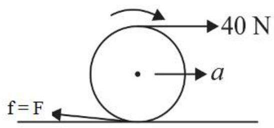
SECTION-2

1.(16) If angular acceleration of the cylinder is α and acceleration of C.O.M. is a, then

$40 - F = ma$ ...(i)

$(40 + F)R = I_{cm}\alpha$ ...(ii)

$a = R\alpha$ ...(iii)



From (ii) and (iii) $\quad 40 + F = I_{cm}\frac{a}{R^2}$ ...(iv)

Adding (i) and (iv)

$80 = \left(\frac{I_{cm}}{R^2} + m\right)a$; $\quad a = \frac{80}{\left(m + \frac{mR^2}{R^2}\right)}$

$\therefore \quad \alpha = \frac{80}{2mR}$ or $\alpha = \frac{80}{2 \times 5 \times 0.5}$; $\quad \alpha = 16$ rad/s^2.

2.(3) For floatation : $m = m_f$

For P, $V_P d_P = \frac{V_P}{2}(1) \Rightarrow d_P = \frac{1}{2}$ g/cm^3

For Q, $V_Q d_Q = \frac{2V_Q}{3}(1) \Rightarrow d_Q = \frac{2}{3}$ g / cm^3

$\therefore \quad \frac{d_P}{d_Q} = \frac{1/2}{2/3} = \frac{3}{4}$

3.(8) Initial energy $E = 4\pi R^2 \times \sigma$

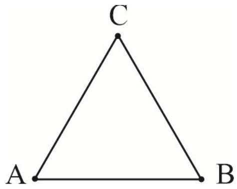
From volume conservation: $\frac{4}{3}\pi R^3 = 729\frac{4}{3}\pi r^3 \quad \Rightarrow \quad R = 9r$

Now final energy $= 729 \times 4\pi r^2\sigma = 9 \times (4\pi R^2)\sigma = 9E$

Work done = Change in surface energy = $9E - E = 8E$

4.(4) $U = U_{AB} + U_{BC} + U_{CA} = -\frac{Gm^2}{l} - \frac{Gm^2}{l} - \frac{Gm^2}{l} = -\frac{3Gm^2}{l}$

When l is change to $3l$, $U' = -\frac{3Gm^2}{3l}$



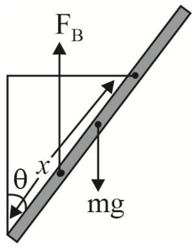
$\therefore$ Work done, $W = U' - U = \frac{-3Gm^2}{3l} + \frac{9Gm^2}{3l} = \frac{6Gm^2}{3l} = \frac{2Gm^2}{l}$

5.(45) Let x be the length of plank inside the water. By applying rotational equilibrium about the hinge, we get

$$mg\frac{l}{2}\sin\theta = F_B\frac{x}{2}\sin\theta \qquad \ldots\text{(i)}$$

If ρ and ρ_0 be densities of plank and water respectively, then

$$mg = \rho A l\,\mathrm{g} \;\; and \;\; F_B = \rho_0\, Axg$$



(A = area of plank)

From Equation (i)

$$\frac{x}{l} = \sqrt{\frac{\rho}{\rho_0}} = \frac{1}{\sqrt{2}}$$

$\Rightarrow$ $x = \frac{1}{\sqrt{2}}m$

Now, $\cos\theta = \frac{0.5}{x} = \frac{1}{\sqrt{2}}$ $\Rightarrow$ $\theta = 45°$

CHEMISTRY

SECTION-1

1.(B) $K_c = \frac{[SO_3]^2}{[SO_2]^2[O_2]}$

$K_c = \frac{1}{[O_2]} = 100;$ $[O_2] = 10^{-2}$

$\frac{n}{10} = 10^{-2} \Rightarrow n = 0.1$

2.(A) $\Delta H = n \times C_p \times \Delta T$ $C_p = C_v + R$

$n = \frac{PV}{RT} = 0.05$ $C_p = 20.794\, JK^{-1}$

For reversible adiabatic $TV^{(\gamma-1)} =$ constant

$T_2V_2^{\gamma-1} = T_1V_1^{\gamma-1}$ $\gamma = 1.66$ for argon

$T_2 = 189.9\,K$ $\Delta T = -110.1K$

Thus $\Delta H = 0.05 \times 20.794 \times -110.1 = -114.47\,J$

3.(D) $\Delta G° = -RT \ln K_P$

$46.5 \times 1000 = -2 \times 298 \times \ln K_P$

$K_P = 1.3 \times 10^{-34}$

Hence, $K_p = (p_{O_2})^{1/2};$ $p_{O_2} = 1.69 \times 10^{-68}$

4.(D) $PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$

on reducing volume to $\frac{V}{2}$, pressure of equilibrium system increase so that equilibrium shift in backward direction and dissociation of PCl_5 decrease.

5.(A) Free energy (G) is a state function.

6.(A) $\Delta H > 0, \Delta S > 0 \Rightarrow$ Reaction may be spontaneous or non-spontaneous.

At 25°C $\Delta G = \Delta H - T\Delta S = 180 - 298 \times 150 \times 10^{-3}$ = 135.3 > 0 = non-spontaneous

To make it spontaneous $(\Delta G < 0)$. We have to increase the temperature.

$T = \frac{\Delta H}{\Delta S} = \frac{180 \times 10^3}{150} = 1200K = 927°C$ (switching temperature)

7.(D) $2AB_3 \rightleftharpoons 2AB_2 + B_2$

	$2AB_3$	$2AB_2$	B_2
t = 0	0.1 mol/lt	0	0
t = eq	0.1 – 0.08	0.08	0.04

Total number of moles per litre = 0.14

$P = \frac{n}{V} \times R \times T = 0.14 \times 0.082 \times 540 = 6.2\,atm$

8.(A) $CaCO_3(s) \rightleftharpoons CaO(s) + CO_2(g)$

$0.2 \qquad 0 \qquad 0$

$0.2 - \frac{0.2 \times 40}{100} \qquad 0.08 \qquad 0.08$

$K_P = p_{CO_2}$

$p_{CO_2} = \frac{0.08 \times 0.082 \times 1067}{10} = 0.69\,atm = 0.7\,atm$

9.(A) At $400K$, $\Delta S_{vap} = \frac{\Delta H_{vap}}{T_b} = 100\,J/mol\,K.$

10.(B) **(i)** Work is a form of exchange energy between system and surrounding in adiabatic process.

(ii) Intensive property is not additive.

(iii) ΔH for cyclic process is zero

(iv) For an isolated system the entropy either increases or remains constant.

11.(A) $\log \frac{K_2}{K_1} = \frac{\Delta H}{R \times 2.303}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$

If $\Delta H = 0$

Then $K_2 = K_1$

Means no effect.

12.(B) For exothermic reaction high temperature favour backward reaction and with increase in pressure reaction goes where number of moles are less.

13.(C) $2A(g) \rightleftharpoons 2B(g) + C(g)$

$v \qquad - \qquad -$

$v - 2v' \qquad 2v' \qquad v'$

$v' = 100$

$v - 2v' + 2v' + v' = 700$

$v = 600$

$2A(g) \rightleftharpoons 2B(g) + C(g)$

$400 \qquad 200 \qquad 100$

$10 \times \frac{400}{700} \qquad 10 \times \frac{200}{700} \qquad 10 \times \frac{100}{700}$

$= \frac{40}{7} \qquad \frac{20}{7} \qquad \frac{10}{7}$

$$K_P = \frac{\left(\frac{20}{7}\right)^2 \times \frac{10}{7}}{\left(\frac{40}{7}\right)^2} = \frac{20 \times 20 \times 10}{40 \times 40 \times 7} = \frac{10}{28}$$

14.(B) $N_2O_4(g) \rightleftharpoons 2NO_2(g);$ $K_P = 4.5$

p

$p - p\alpha$ $2p\alpha$

Total pressure at equilibrium $= p - p\alpha + 2p\alpha = 2$ $\Rightarrow$ $p + p\alpha = 2$

Hence $K_P = \frac{[p_{NO_2}]^2}{[p_{N_2O_4}]}$ $\Rightarrow$ $4.5 = \frac{4p^2\alpha^2}{p(1-\alpha)}$

Hence, $\alpha = 0.6$

$1+\alpha = \frac{\text{molar mass (initial)}}{\text{molar mass (average)}}$; $1+\alpha = \frac{92}{\text{molar mass (average)}}$

Molar mass (average) $= \frac{92}{1+0.6} = \frac{92}{1.6} = 57.5$

15.(B) $\Delta C_p = 2\times 25.1 + 3\times 75.3 - (103.8 + 3\times 28.8) = 85.9\,J/mole$

$\frac{\Delta H_{358} - \Delta H_{298}}{T_2 - T_1} = \Delta C_p$ $\Rightarrow$ $\Delta H_{358} = -28.136\,kJ/mole$

16.(B) As $PV^\gamma = \text{constant}$ $\Rightarrow$ $P\cdot\gamma v^{\gamma-1}\times dv + V^\gamma \cdot dp = 0$

$\frac{dP}{P} = -\gamma\cdot\frac{V^{\gamma-1}}{V^\gamma}dv$ $\Rightarrow$ $\frac{dp}{P} = -\gamma\frac{dV}{V}$

17.(C) $2CH_3OH(\ell) + 3O_2 \longrightarrow 4H_2O(\ell) + 2CO_2(g) + 1453$

$4\times 44 + 4H_2O(\ell) \longrightarrow 4H_2O(g)$

$2CH_3OH(\ell) + 3O_2(g) \longrightarrow 4H_2O(g) + 2CO_2(g) + 1277;$ $\Delta H = -1277kJ$

18.(A) 1 watt = 1 J/sec

Total heat for 36 mL $H_2O = 806\times 100 = 80600\,J$

$\Delta H_{vaporisation} = \frac{80600}{36}\times 18 = 40300\,J$ or $40.3\,kJ/mole$

19.(B) For gas A the temperature remains constant but in case of gas B the temperature increases so the pressure increases. Therefore, the final pressure of 'A' will be less than that of B.

20.(D) $NO + NO_3 \rightleftharpoons 2NO_2$

	NO	NO_3	$2NO_2$
$t = 0$	1	3	0
$t = t_{eq}$	$1-\frac{x}{2}$	$3-\frac{x}{2}$	x
$t' = t'_{eq}$	$3-x$	$3-x$	$x+x$

$K = \frac{x^2}{\left(1-\frac{x}{2}\right)\left(3-\frac{x}{2}\right)} = K = \frac{4x^2}{(3-x)^2}$

Equating the two gives $x = \frac{3}{2}$ then $K = 4$

SECTION-2

1.(3) $2X(g) \rightleftharpoons 2Y(g) + Z(g)$

$1-\alpha \qquad \alpha \qquad \frac{\alpha}{2}$

Total moles $= 1+\frac{\alpha}{2} = \frac{2+\alpha}{2}$

$$K_P = \frac{(p_Y)^2 (p_Z)}{(p_x)^2} = \frac{\left(\frac{2\alpha}{(2+\alpha)}\right)^2 P_t^2 \cdot \left(\frac{\alpha}{2+\alpha}\right) P_t}{\left(\frac{2(1-\alpha)}{(2+\alpha)}\right)^2 P_t^2} \quad \text{(Neglecting } \alpha \text{ compared to 1)}$$

$$\alpha = \left(\frac{2K_P}{P_t}\right)^{1/3} \Rightarrow \quad n = 3$$

2.(1) Slope $= \frac{-\Delta H}{2.303R}$

So, $\frac{-\Delta H}{2.303R} = \frac{-1}{4.606}$; $\Delta H = 1\,\text{cal}$

3.(89) $\underset{N\equiv N}{N_2} + \frac{1}{2}\underset{O=O}{O_2} \longrightarrow N = N = O\,(N_2O)$

$$\Delta_r H = \Sigma BE_R - \Sigma BE_P = \left(946.2 + \frac{497}{2}\right) - (418 + 605.3) = 171.4\,\text{kJ/mol}$$

Resonance energy $= \Delta H_{exp} - \Delta H_{cal} = 82.4 - 171.4 = -89\,\text{kJ}$

4.(4) $\frac{1}{2}H_2(g) + \frac{1}{2}Cl_2(g) \rightleftharpoons HCl(g),\ \Delta G^\circ = 1.72\text{kJ mol}^{-1}$

ΔG° for $2HCl(g) \rightleftharpoons H_2(g) + Cl_2(g)$ is $[2\times(-1.72)]\text{kJ}$

$\Delta G^\circ = -2.303RT\ \log K_p$

$-3.44\times10^3 = -5700\log K_p$

$\log K_p = 0.6 \quad \Rightarrow \quad K_p = 4$

5.(8) $A(g) + 3B(g) \rightleftharpoons 4C(g)$

$a-x \qquad a-3x \qquad 4x$

Here, $(a - x = 4x \quad \Rightarrow \quad a = 5x)$

$[A] = a - x = 5x - x = 4x$

$[B] = a - 3x = 5x - 3x = 2x$

$[C] = 4x$

$$K_c = \frac{[C]^4}{[A]\times[B]^3} = \frac{(4x)^4}{4x\times(2x)^3} = 8$$

MATHEMATICS

SECTION-1

1.(C) $\left|\dfrac{2(3a+4b+c)+3(4a+3b+c)}{\sqrt{a^2+b^2}}\right|=10$

$\Rightarrow \left|\dfrac{18a+17b+5c}{\sqrt{a^2+b^2}}\right|=10 \Rightarrow \left|\dfrac{\frac{18}{5}a+\frac{17}{5}b+c}{\sqrt{a^2+b^2}}\right|=2$

$\Rightarrow\ ax+by+c=0$

Touches circle with center $\left(\dfrac{18}{5},\dfrac{17}{5}\right)$ and radius = 2

2.(D) The equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents the general equation of pair of lines which are parallel to each other, then the distance between them is given by,

$d=\left|2\sqrt{\dfrac{g^2-ac}{a(a+b)}}\right|$

Here, the equation is,

$x^2+2xy+y^2-8ax-8ay-9a^2=0$

i.e., $a=1, b=1,\ c=-9a^2, h=1,\ f=-4a,\ g=-4a$

$\because\ d=25\sqrt{2}$

$\Rightarrow\quad 25\sqrt{2}=\left|2\sqrt{\dfrac{(-4a)^2-1(-9a^2)}{1(1+1)}}\right| \quad\Rightarrow\quad 25\sqrt{2}=\left|2\sqrt{\dfrac{16a^2+9a^2}{2}}\right| \quad\Rightarrow\quad 25\sqrt{2}=\sqrt{2}\,|5a|$

3.(B) $A=\dfrac{1}{2}\begin{vmatrix}0 & 0 & 1\\ 0 & \dfrac{c}{b} & 1\\ \dfrac{c}{a} & 0 & 1\end{vmatrix}=\dfrac{1}{2}\left|\left[0-\dfrac{c^2}{ab}\right]\right|\Rightarrow A=\dfrac{c^2}{2ab}$

$c^2=ab\Rightarrow a, c, b$ are G.P.

4.(A) Using $T=S_1$ take mid-point (h, k) then equation of PQ is

$xh+yk-\left(h^2+k^2\right)=0$ …(i)

$mx-y+2m=0$ …(ii)

Compare (i) and (ii) eliminate m

$\dfrac{h}{m}=\dfrac{-k}{1}=\dfrac{-\left(h^2+k^2\right)}{2m} \qquad \Rightarrow \dfrac{-h}{k}=m$

$-2m\,k=h^2+k^2$

$\Rightarrow -2h=h^2+k^2\ \Rightarrow x^2+y^2+2x=0$

5.(C) The given lines will be parallel to lines $ax^2 - 6xy + y^2 = 0$

So, $\left(\frac{y}{x}\right)^2 - 6\left(\frac{y}{x}\right) + a = 0$

$m + m^2 = 6$

Which gives $m = -3\, or\, 2$ … (1)

and $mm^2 = a$

From (2) a = –27 or 8 … (2)

Hence sum of all possible value of a = – 19

6.(A) Slope of reflected ray = $\frac{3}{4} \Rightarrow$ slope of incident ray $= -\frac{3}{4}$

$\Rightarrow$ Equation of incident ray is $(y+4) = -\frac{3}{4}(x+2)$ i.e., $4y + 3x + 22 = 0$

7.(A) Let $A(\alpha, 0)$ and $B(\beta, 0)$ be the two points

$OT^2 = OA.OB = \alpha\beta = \frac{c}{a}$

8.(B) $\frac{|PA| + |PB|}{2} > \left[|PA|.|PB|\right]^{1/2}$

$|PA| + |PB| > 2|PT| = 2\sqrt{3}$

Maximum length occurs when PAB passes through center

i.e. $|PA| + |PB| = 4$ (Maximum)

So, range is $\left(2\sqrt{3}, 4\right]$

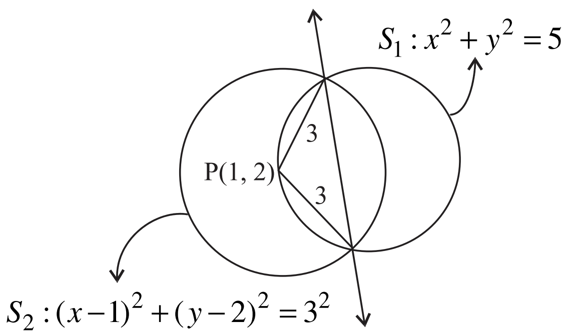
9.(B) Let $(\alpha, 3-\alpha)$ be any point on x + y = 3

$\therefore$ Equation of chord of contact is $\alpha x + (3-\alpha)y = 9$

i.e., $\alpha(x-y) + 3y - 9 = 0$

The chord passes through the point (3, 3) for all values of α

10.(B)



Equation of common chord AB is given by:

$S_1 - S_2 = 0$

$2x + 4y - 1 = 0$

11.(A) L_1 is direct common tangent of these circles

As A_1 & A_2 lie on different sides of L_1 hence, L_1 is transverse common tangent

Radius of $C_1 = \left|\frac{3-8+1}{5}\right| = \frac{4}{5}$

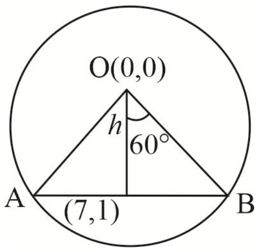
Radius of $C_2 = \left|\frac{9-4+1}{5}\right| = \frac{6}{5}$

12.(C) Let the chord is AB which subtends and angle θ at the center (0, 0)

$\Rightarrow \quad \theta + 2\theta = 360° \quad \Rightarrow \quad \theta = 120° = \angle AOB$

Let the distance of O from AB = h

Then, $\cos 60° = \frac{h}{10} = \frac{1}{2} \Rightarrow h = 5$



Let the equation of the chord is $\frac{y-1}{x-7} = m$

$\Rightarrow \quad mx - y + 1 - 7m = 0$ whose distance from (0, 0) is equal to 5

$\Rightarrow \quad \left|\frac{0-0+1-7m}{\sqrt{1+m^2}}\right| = 5$

$\Rightarrow \quad 1 - 14m + 49m^2 = 25 + 25m^2$

$\Rightarrow \quad 24m^2 - 14m - 24 = 0 \Rightarrow m_1 m_2 = -1$

$\Rightarrow$ Chords are perpendicular

13.(C) Vertices are $A(a, 0)$, $B(-a, 0)$ and $C(b, c)$

$\therefore$ Centroid is $G\left(\frac{b}{3}, \frac{c}{3}\right)$

$$\frac{AB^2 + BC^2 + CA^2}{GA^2 + GB^2 + GC^2} = \frac{4a^2 + (a+b)^2 + c^2 + (a-b)^2 + c^2}{\left(\frac{b}{3}-a\right)^2 + \left(\frac{c}{3}\right)^2 + \left(\frac{b}{3}+a\right)^2 + \left(\frac{c}{3}\right)^2 + \left(\frac{2b}{3}\right)^2 + \left(\frac{2c}{3}\right)^2} = \frac{4a^2 + 2c^2 + 2a^2 + 2b^2}{\frac{2b^2}{9} + 2a^2 + \frac{6c^2}{9} + \frac{4b^2}{9}} = 3$$

Note: we can also solve this faster geometrically by using Apollonius theorem which gives length of median in terms of length of 3 sides of triangle and use the fact that AG = 2/3 AD, where AD is the length of median from A etc.

14.(A) $(a + b\lambda)x + (2b - 2a\lambda)y + (3b - 3\lambda a) = 0$

$\because \quad a + b\lambda = 0 \Rightarrow \lambda = -\frac{a}{b} \qquad \therefore \quad y = -\frac{3}{2}$

15.(B) For x-intercept put y = 0

$2x^2 - 4x - 6 = 0$

$\Rightarrow x^2 - 2x - 3 = 0 \rightarrow x_1, x_2$; $\quad x_1 + x_2 = 2$, $\quad x_1 x_2 = -3$

x intercept = $|x_1 - x_2| = \sqrt{(x_1+x_2)^2 - 4x_1 x_2}$

$= \sqrt{4+12} = \sqrt{16} = 4$

16.(D) Equation of normal

$y-5=m(x-3)$

If it is tangent to $x^2+y^2=9$ then

$\left|\frac{3m-5}{\sqrt{1+m^2}}\right|=3 \Rightarrow m=\frac{8}{15}$, Not defined

17.(B) The given relation is $(5a-4b)^2-c^2=0$

$\Rightarrow \quad (5a-4b+c)(5a-4b-c)=0$

$\Rightarrow \quad \left[2a\left(\frac{5}{2}\right)+b(-4)+c\right]\left[2a\left(-\frac{5}{2}\right)+b(4)+c\right]=0$

$\Rightarrow$ The line $2ax+by+c=0$ passes through $\left(\frac{5}{2},-4\right)$ or $\left(-\frac{5}{2},4\right)$

18.(B) We have, $\text{Area}(\Delta OPQ)=\frac{1}{2}\begin{vmatrix}0 & 0 & 1\\ a & \frac{2a}{3} & 1\\ b & \frac{-2b}{3} & 1\end{vmatrix}=5$ (Given) $\Rightarrow \frac{4ab}{3}=\pm 10$

So, $4ab=\pm 30$ …… (i)

Also $2h=a+b$ …… (ii)

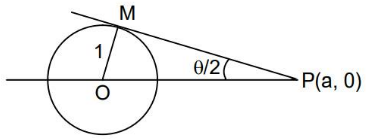
and $2k=\frac{2a-2b}{3} \Rightarrow a-b=3k$ …… (iii)

As $4ab=(a+b)^2-(a-b)^2 \Rightarrow \pm 30=4h^2-9k^2$ [Using (i), (ii) and (iii)]

So required locus is $4x^2-9y^2=\pm 30$

19.(A) $\frac{\pi}{3}<\theta<\pi$

$\Rightarrow \frac{\pi}{6}<\frac{\theta}{2}<\frac{\pi}{2} \Rightarrow \frac{1}{2}<\frac{1}{a}<1 \Rightarrow 1<a<2$

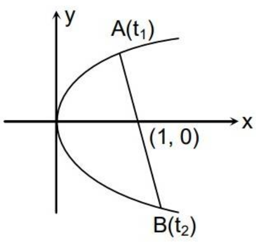


$\therefore$ $a \in (-2,-1)\cup(1,2)$

20.(D) $m_{AB}=\frac{2}{t_1+t_2}=1$

$t_1+t_2=2$

$t_1t_2=-1$

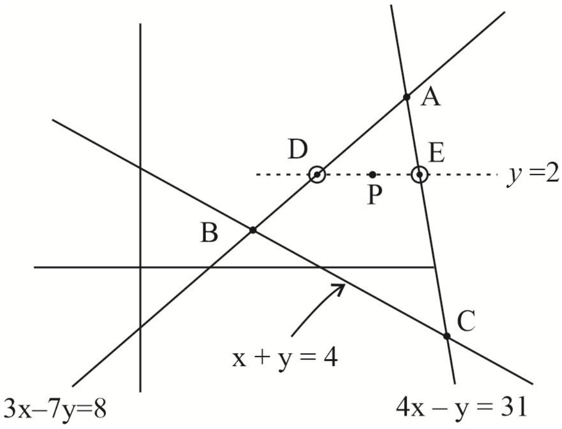


$\therefore$ Required equation is $m^2+2m-1=0$

SECTION-2

1.(1) Let $P=(\lambda, 2)$

First roughly draw ΔABC. The point $P(\lambda, 2)$



Move on the line $y = 2$ for all

Now D and E are the intersection of $3x - 7y = 8$,

$y = 2$ and $4x - y = 31$, $y = 2$ respectively

$\therefore \quad D=\left(\frac{22}{3},2\right)$ and $E=\left(\frac{33}{4},2\right)$ i.e., $\lambda \in\left(\frac{22}{3},\frac{33}{4}\right)$

2.(0) $x^3-x^2-x-2=0 \Rightarrow (x-2)\left(x^2+x+1\right)=0 \Rightarrow x=2$ (i)

$xy^2+2xy+4x-2y^2-4y-8=0 \Rightarrow (x-2)\left(y^2+2y+4\right)=0 \Rightarrow x=2$ (ii)

Both the equations represent same line. So, number of triangles formed are zero.

3.(2) Equation of tangent for $x^2=4y$

$x=\frac{1}{m}y+m$

$\therefore$ It is a tangent to $xy = -2$

$\therefore \quad \left(\frac{1}{m}y+m\right)y=-2$

$\Rightarrow \quad \frac{1}{m}y^2+my+2=0$

$\therefore \quad m^2-4\cdot\frac{1}{m}\cdot 2=0 \Rightarrow m^3=8$

$\therefore \quad m=2$

4.(32) Since circle touches both the axes and passes through (4, 4), it lies in the first quadrant, so its equation is $(x-r)^2+(y-r)^2=r^2$

(4, 4) lies on it hence $r^2-16r+32=0$

Hence, the product of roots (radii) = 32

5.(8) HM of SP and $\mathrm{SQ}=\frac{2(3)(6)}{3+6}=4=$ semi latus rectum.